P. Vojta   # Math 1A First Midterm   50 minutes
Fall 2001

1. (16 points) Find the following limits:

   (a). $\displaystyle \lim_{x \to 3} \frac{x^2 - 5x + 6}{x^2 - 9}$

   (b). $\displaystyle \lim_{x \to 3-} \frac{\ln x}{x - 3}$

   (c). $\displaystyle \lim_{x \to 0+} \frac{(\ln x)^2 + 1}{(\ln x)^2 + 3}$

   (d). $\displaystyle \lim_{x \to 0} \sin\left(\frac{\pi |x|}{x}\right)$

   (e). $\displaystyle \lim_{x \to \infty} \frac{10 + \sin x}{x(1 + \sin x)}$

2. (6 points) Find

$$\lim_{x \to \infty} \frac{1}{x + \frac{1}{\ln x}},$$

   using *only* the Squeeze Theorem and other theorems from the book (including limit laws explicitly given in the book, but not informal arguments such as "this is big and that is big, so their product is big"). Explicitly mention those theorems and limit laws that you use. (If you use limit laws, you may give them by name instead of by number, and you do not need to explicitly justify steps from high-school algebra.)

3. (11 points) Find the following derivatives:

   (a). $\displaystyle \frac{d}{dx}(x^7 + \frac{9}{5}x^5 - x^3 - 2.3)$

   (b). $\displaystyle \frac{d}{dx}(x^2 e^x \sin x \tan x)$

   (c). $\displaystyle \frac{d}{dx}\left(\frac{\csc x}{x^2 + x + 1}\right)$

4. (6 points) Let

$$f(x) = \begin{cases} \frac{1}{2}x^2 & \text{if } -1 \le x \le 1; \\ x - 1 & \text{if } 1 < x \le 2; \text{ and} \\ \frac{1}{3-x} & \text{if } 2 < x < 3. \end{cases}$$

   Find the intervals where f is continuous, and the intervals where f is differentiable. Express your answer as concisely as possible, and explain your reasons.

   (a). Intervals where f is continuous:

   (b). Intervals where f is differentiable:

5. (6 points) Find the following derivative using the *definition* of the derivative (i.e., using a limit):

$$\frac{d}{dx}\sqrt{1 + 2x}$$

6. (5 points) Find the equation of the tangent line to the curve $y = x^4$ at the point $(2, 16)$.